9.1 Developing Null and Alternative Hypotheses

- Hypothesis testing can be used to determine whether a statement about the value of a population parameter should or should not be rejected.

- The null hypothesis $H_0$ is a tentative assumption about a population parameter.
  - Usually represents a state of no change or no difference from the experiment’s point of view. It is assumed to be true until sufficient evidence is obtained to warrant its rejection.

- The alternative hypothesis, denoted by $H_a$, is the opposite of what is stated in the null hypothesis.
  - Is the statement on which the researcher places the burden of proof substantial supporting evidence is required before it will be accepted.

- The hypothesis testing procedure uses data from a sample to test the two competing statements indicated by $H_0$ and $H_a$.

- How the null and alternative hypotheses should be formulated is not always obvious.
Guidelines for establishing the null and alternative hypotheses for common situations in which hypothesis testing procedures are used

**Testing Research Hypotheses**

- As a general guideline, a research hypothesis should be stated as the alternative hypothesis.
- In research studies, the null and alternative hypotheses should be formulated so that the rejection of $H_0$ supports the research conclusion.
- The conclusion that the research hypothesis is true comes from sample data that contradict the null hypotheses.

**Testing the Validity of a Claim**

- In any situation that involves testing the validity of a claim, the null hypothesis is generally based on the assumption that the claim is true.
- The alternative hypothesis is then formulated so that rejection of $H_0$ will provide statistical evidence that the stated assumption is incorrect.
- The conclusion that the claim is false comes from sample data that contradict the null hypothesis.
Testing in Decision-Making Situations

• In this case action must be taken both when $H_0$ cannot be rejected and when $H_0$ can be rejected.

• In general, this type of situation occurs when a decision maker must choose between two courses of action, one associated with the null hypothesis and another associated with the alternative hypothesis.

Summary of Forms for Null and Alternative Hypotheses

• The equality part of the hypothesis always appears in the null hypothesis.

• In general a hypothesis test about the value of the population mean $\mu$ must take on one of the following three forms (where $\mu_o$ is the hypothesized value of the population mean)

  $H_0: \mu \geq \mu_o$  \hspace{1cm} $H_0: \mu \leq \mu_o$  \hspace{1cm} $H_0: \mu = \mu_o$

  $H_a: \mu < \mu_o$  \hspace{1cm} $H_a: \mu > \mu_o$  \hspace{1cm} $H_a: \mu \neq \mu_o$

  One-tailed  \hspace{1cm} One-tailed  \hspace{1cm} Two-tailed

• The first two forms are called a one-tailed test and the third is called a two-tailed test.

• The null and alternative hypotheses are competing statements about the population. Either the null hypothesis is true or the alternative hypothesis is true, but not both.
9.2 Type I and Type II Errors

- Since hypothesis tests are based on sample data, we must allow for the possibility of errors.
- A **Type I error** is rejecting $H_0$ (null hypothesis) when it is true. This is the more crucial error.
- A **Type II error** is accepting $H_0$ (null hypothesis) when it is false. This is the less crucial error.
- The person conducting the hypothesis test specifies the maximum allowable probability of making a Type I error, denoted by $\alpha$ and called the **level of significance**.
  - We can control this value.
- Generally, we can not control for the probability of making a Type II error, denoted by $\beta$.
- We side-step the risk of making a Type II error by using the term “do not reject $H_0$” instead of “accept $H_0$”.

9.3 Population Mean: $\sigma$ Known

- Methods presented in this section are exact if the sample is selected from a population that is normally distributed. In cases where the population is not normally distributed, these methods are still applicable if the sample size is large enough.
One-Tailed Test:

- One-tailed tests about a population mean take on one of the following two forms:
  
  **Lower Tail Test**  
  \( H_0: \mu \geq \mu_0 \)  
  \( H_a: \mu < \mu_0 \)  

  **Upper Tail Test**  
  \( H_0: \mu \leq \mu_0 \)  
  \( H_a: \mu > \mu_0 \)  

- For hypothesis tests about a population mean for the \( \sigma \) known case, we use the standard normal random variable \( z \) as a test statistic to determine whether \( \bar{x} \) deviates from the hypothesized value \( \mu_0 \) enough to justify rejecting the null hypothesis and thus the acceptance of the alternative

- Key Question for a lower tail test: How small must the test statistic \( z \) be before we choose to reject the null hypothesis?

- Two approaches can be used to answer this question
  - P-value approach
  - Critical value approach

P-Value Approach

- Preferred method of determining whether the null hypothesis can be rejected

- The p-value is the probability of obtaining a sample result that is at least as unlikely as what is observed. (ranges from 0 to 1)
The p-value can be used to make the decision in the hypothesis test by noting that:

- If the p-value is less than the level of significance $\alpha$, the value of the test statistic is in the rejection region.
- If the p-value is greater than or equal to the level of significance $\alpha$, the value of the test statistic is not in the rejection region.

**Critical Value Approach**

- For a lower tail test, the critical value is the value of the test statistic that corresponds to an area of $\alpha$ (level of significance) in the lower tail of the sampling distribution of the test statistic. (p. 347)
- The test statistic $z$ has a standard normal probability distribution.
- We can use the standard normal probability distribution table to find the $z$-value with an area of $\alpha$ in the lower (or upper) tail of the distribution.
- The value of the test statistic that established the boundary of the rejection region for the test is called the critical value for the test.

**Two-Tailed Test:**

- General form for a two-tailed test about a population mean is as follows:
\[ H_0: \mu = \mu_0 \]
\[ H_a: \mu \neq \mu_0 \]

Test Statistic for Hypothesis Test about a Population Mean: \( \sigma \) Known (same as one-tailed test)

**P-Value Approach**

- Compute the value of the test statistic \( z \)
- If the value of the test statistic is in the upper tail \( (z > 0) \), find the area under the standard normal curve to the right of \( z \)
- If the value of the test statistic is in the lower tail, find the area under the standard normal curve to the left of \( z \)
- Double the tail area, or probability, obtained in either step 2 or 3 to obtain the p-value

**Critical Value Approach**

- Critical values for a two-tailed test will occur in both the lower and upper tails of the standard normal distribution

**Steps of Hypothesis Testing**

- Determine the appropriate hypothesis
- Specify the level of significance
- Collect the sample data and compute the value of the test statistic
• A) Critical Value Approach
  ○ Compare the test statistic to the critical values in the rejection rule

Or

B) P-Value Approach
  ○ Compute the p-value based on the test statistic and compare it to $\alpha$, to determine whether or not to reject $H_0$.

9.4 Population Mean: $\sigma$ Unknown
• The $\sigma$ unknown case corresponds to situations in which an estimate of the population standard deviation cannot be developed prior to sampling, the sample must be used to develop an estimate of both $\sigma$ and $\mu$.
• To conduct hypothesis test about a population mean for the $\sigma$ unknown case, the sample mean $\bar{x}$ is used as an estimate of $\mu$ and the sample standard deviation $s$ is used as an estimate of $\sigma$.
• Steps of the hypothesis testing procedure for the $\sigma$ unknown case are the same as those for the $\sigma$ known case.
• With $\sigma$ unknown the computation of the test statistic and p-value are a little different
• With the \( \sigma \) unknown case the sampling distribution of the test statistic has more variability than with the \( \sigma \) known case because the sample is used to develop both \( \mu \) and \( \sigma \) so we use a t distribution

**Confidence Interval Approach to a Two-Tailed Test about a Population Mean**

• Select a simple random sample from the population and use the value of the sample mean \( \bar{x} \) to develop the confidence interval for the population mean \( \mu \)
• If the confidence interval contains the hypothesized value \( \mu_0 \) do not reject \( H_0 \). Otherwise reject \( H_0 \)

**p-values and t distribution**

• The format of the t distribution is most statistics textbooks does not have sufficient detail to determine the exact p-value for a hypothesis test
• However, we can still use the t distribution to identify a range for the p-value

**9.5 Population Proportion**

• Let \( P_0 \) denote the hypothesized value for the population proportion
• The equality part always appears in the null hypothesis
• Hypothesis tests about a population proportion are based on the difference between the sample proportion \( \bar{p} \) and the hypothesized population proportion \( P_0 \).
• The methods used to conduct a hypothesis test are similar to those used for hypothesis tests about a population mean
  o Only difference is we use the sample proportion and its standard error to compute the test statistic
• The p-value or critical value approach is used to determine whether the null hypothesis should be rejected
• The sampling distribution of \( \bar{p} \), the point estimator of the population parameter \( p \), is the basis for developing the test statistic
• In chapter 7 we learned that if \( np \geq 5 \) and \( n(1-p) \geq 5 \) the sampling distribution of \( \bar{p} \) can be approximated by a normal distribution (almost all cases)
• So the standard normal random variable \( z \) is the test statistic used to conduct hypothesis tests about a population proportion

9.6 Hypothesis Testing and Decision Making
• In many decision making situations the decision maker may want, and in some cases be forced, to take action with both the conclusion do not reject \( H_0 \) and the conclusion reject \( H_0 \)
• When testing research hypotheses or testing the validity of a claim, action is taken only when the null hypothesis is rejected and the alternative hypothesis is concluded to be true.

• When testing in decision-making situations it is necessary to take action when the null hypothesis is not rejected as well as when it is rejected.

• In such decision-making situations it is recommended that the hypothesis testing procedure be extended to include consideration of making a Type II error.

9.7 Calculating the Probability of Type II Errors

The following step-by-step procedure can be used to compute the probability of making a Type II error in hypothesis tests about a population mean:

• Formulate the null and alternative hypotheses
• Use the level of significance \( \alpha \) to establish a rejection rule based on the test statistic
• Using the rejection rule solve for the value of the sample mean that identifies the rejection region
• Use the results from step 3 to state the values of the sample mean that lead to the acceptance of Ho: this defines the acceptance region

• Using the sampling distribution of \( \bar{x} \) for a value of \( \mu \) satisfying the alternative hypothesis, and the acceptance region from step 4, to compute the probability that the sample mean will be in the acceptance region. This probability is the probability of making a Type II error at the chosen value of \( \mu \)

**Power of the Test**

• The probability of correctly rejecting \( H_0 \) when it is false is called the power of the test (p.373)

• For any particular value of \( \mu \) the power is \( 1 - \beta \) (p.373)

• We can show graphically the power associated with each value of \( \mu \); such a graph is called a power curve

**9.8 Determining the Sample Size for Hypothesis Tests about a Population Mean**

**Relationship between \( \alpha \), \( \beta \), and \( n \)**

• Once two of the three values are known, the other can be computed

• For a given level of significance \( \alpha \), increasing the sample size \( n \) will reduce \( \beta \)

• For a given sample size \( n \), decreasing \( \alpha \) will increase \( \beta \), whereas increasing \( \alpha \) will decrease \( \beta \)